Bruhat Tits tree and its Nekrashevych-Röver group

Thompson group V

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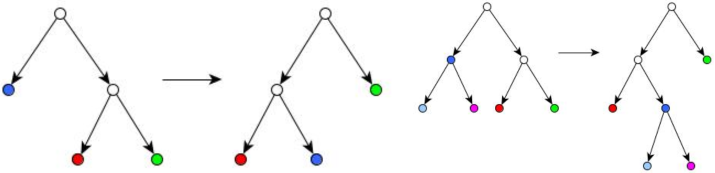

Figure: Example of an element of V

On figure we performed *simple expansion* at the blue vertex.

Colour preserving Thompson groups

We can extend this definition on a class of coloured trees:
We take set of colours with production rule = rule how to colour offsprings. Then Consider coloured rooted tree T which obeys the production rule, we call such tree self similar tree. We can define in similar way group V_T. This time we require leaf bijections to be colour-preserving.

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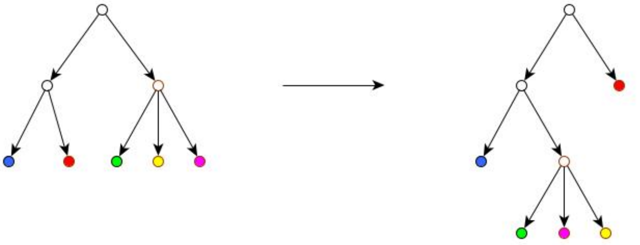

Figure: Here we have two colours clack and brown with production rules black - (black,brown) and brown - (black, brown, brown)

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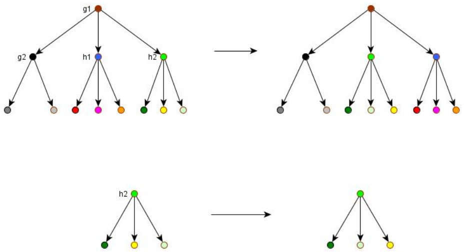

In a self similar tree for each edge e of colour c and each g in G_c we get natural element of tree ends permutation group. We call it $\varphi(g, e)$.

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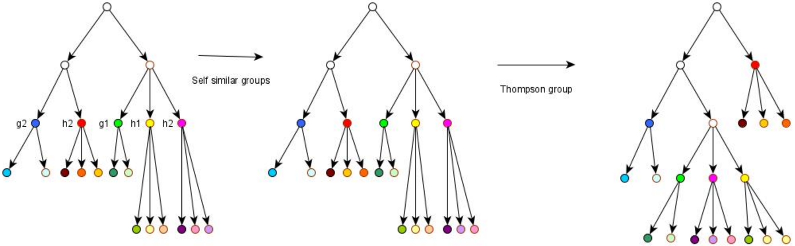

Bruhat Tits tree

Consider $\mathbb{Q}_p$ p-adic numbers and associated lattices We can define equivalence relation on lattices $L_1 \ \ L_2$ iff $L_1 = p^k L_2$ for some integer k. We can define relation R on lattices by $L_1 R L_2$ iff $L_1 < L_2 < pL_1$. Adapt this to be a symmetric relation on above equivalence classes and then associated graph will be $p+1$ regular tree. Hence there is an action of $PGL_2(\mathbb{Q}_p)$, and this action is transitive.

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Permutation expansion of $M \in PGL_2(\mathrm{Q}_p)$ at a rigid vertex give rise to a permutation of offsprings which corresponds to a Möbius transformation of M on PF_p (after identification of offsprings with point in PF_p).

Derived subgroups of NR groups

Take quotient map into abelianization for each edge stabilizer and V_T and put these together. Quotient the image by relations arising from permutation expansions. Kernel of this map is the derived subgroup. It can be shown that the derived subgroup is simple (cf. the derived subgroup of V_t is simple).

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For Bruhat Tits tree we get abelianizations of edge stabilizers being $\mathbb{F}_p^\times$. R identifies g at rigid vertives with g^{-1} in barycentric subdivisions. Contribution from V_T' is related to the sign of permutations indeuced from Möbius transformations.